

## UBC Math Circle 2023 Problem Set 2

**Problem 1.** Let  $p(x) = x^2 - 3x + 2$ . Show that for any positive integer  $n$  there exist unique numbers  $a_n, b_n$  such that the polynomial  $q_n(x) = x^n - a_nx - b_n$  is divisible by  $p(x)$ .

**Problem 2.** Find all functions  $f : \mathbb{N} \rightarrow \mathbb{N}$  satisfying

$$f(f(f(n))) + 6f(n) = 3f(f(n)) + 4n + 2001, \forall n \in \mathbb{N}$$

**Problem 3.** A polygon is called *convex* if all its internal angles are smaller than 180 degrees. Given a convex polygon, prove that one can find three distinct vertices  $A, P, Q$ , where  $PQ$  is a side of the polygon, such that the perpendicular from  $A$  to the line  $PQ$  meets the segment  $PQ$ .