UBC Math Circle 2023 Problem Set 8

1. An ordered triple of numbers is given. It is permitted to perform the following operation on the triple: to change two of them, say a and b, to $\frac{a+b}{\sqrt{2}}$ and $\frac{a-b}{\sqrt{2}}$. Is it possible to obtain the triple $(1, \sqrt{2}, 1 + \sqrt{2})$ from the triple $(2, \sqrt{2}, \frac{1}{\sqrt{2}})$ using this operation?

2. Given n points in the plane, no three of which are collinear, show that there exists a closed polygonal line with no self-intersections having these points as vertices.

3. Prove that the figure eight knot in the Figure is knotted.