

UBC MATH CIRCLE 2024 PROBLEM SET 1

Problem 1. Find all pairs of integers (a, n) for which the following holds.

$$\frac{(a+1)^n - a^n}{n} \in \mathbb{Z}.$$

Problem 2. Find all integers x and y for which $x^3 - y^2 = 9$. (As a bonus problem, what happens when 9 is replaced by 7?)

Problem 3. Find all polynomials $f \in \mathbb{R}[x]$ such that for all real numbers a, b, c satisfying $ab + bc + ca = 0$, we have

$$f(a-b) + f(b-c) + f(c-a) = 2f(a+b+c).$$