UBC MATH CIRCLE 2024 PROBLEM SET 10

Problem 1. *Determine all roots of the system of equations*

$$x + y = z = 3$$
$$x^2 + y^2 + z^2 = 3$$
$$x^3 + y^3 + z^3 = 3$$

Problem 2. *Find all polynomials $P(x)$ with real coefficients such that $P(x)P(x+1) = P(x^2)$ for all $x \in \mathbb{R}$.*

Problem 3. *A rectangle $\mathcal{R}$ with odd integer side lengths is divided into small rectangles with integer side lengths. Prove that there is at least one of the small rectangles whose distances from the four sides of $\mathcal{R}$ are either all odd or all even.*

Date: April 19, 2024.