UBC MATH CIRCLE 2024 PROBLEM SET 2

Problem 1. *Let $f \in \mathbb{Z}[x]$ be a polynomial with integer coefficients and let $S \subset \mathbb{Z}$ be a finite set of positive integers such that for any $n \in \mathbb{Z}$, there is a $s \in S$ such that $s \mid f(n)$. Show that there is an $s \in S$ such that $s \mid f(n)$ for all $n \in \mathbb{N}$.*

Problem 2. *Is it possible to cover the plane with the interiors of a finite number of parabolas?*

Problem 3. *Let $\mathbb{Z}^n$ be the integer lattice in $\mathbb{R}^n$. Two points in $\mathbb{Z}^n$ are called* neighbors *if they differ by exactly 1 in one coordinate and are equal in all other coordinates. For which integers $n \geq 1$ does there exist a set of points $S \subset \mathbb{Z}^n$ satisfying the following two conditions?*

(1) *If p is in S, then none of the neighbors of p is in S.*
(2) *If $p \in \mathbb{Z}^n$ is not in S, then exactly one of the neighbors of p in S.*

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