# UBC MATH CIRCLE 2024 PROBLEM SET 5

**Problem 1.** *Basketball star Shanille OKeal's team statistician keeps track of the number, $S(N)$, of successful free throws she has made in her first $N$ attempts of the season. Early in the season, $S(N)$ was less than $80\%$ of $N$, but by the end of the season, $S(N)$ was more than $80\%$ of $N$. Was there necessarily a moment in between when $S(N)$ was exactly $80\%$ of $N$?*

**Problem 2.** *Recall that a regular icosahedron is a convex polyhedron having $12$ vertices and $20$ faces; the faces are congruent equilateral triangles. On each face of a regular icosahedron is written a nonnegative integer such that the sum of all $20$ integers is $39$. Show that there are two faces that share a vertex and have the same integer written on them.*

**Problem 3.** *Let $f(x) = 3x^2 + 1$. Prove that for any positive integer $n$, the product $f(1)f(2)\dots f(n)$ has at most $n$ prime divisors. (Bonus: Show that for $n \geq 4$ it has at most $n-1$ prime divisors).*

*Date*: February 25, 2024.