UBC MATH CIRCLE 2024 PROBLEM SET 7

Problem 1. *Let f be a real-valued function on the plane such that for every square $ABCD$ in the plane, $f(A) + f(B) + f(C) + f(D) = 0$. Does it follow that $f(P) = 0$ for all P in the plane?*

Problem 2. *Let m be positive integers $a_1, \ldots, a_m$ be given. Prove that there exist fewer than 2^m positive integers $b_1, \ldots, b_n$ such that all sums of distinct b_k's are distinct and all a_i for $1 \leq i \leq m$ occur among them.*

Problem 3. *For any polynomial $P \in \mathbb{C}[x]$ and for each complex number a, denote by P_a the set of all $z_0 \in \mathbb{C}$ such that $P(z_0) = a$. Let $P, Q \in \mathbb{C}[x]$ such that $P_2 = Q_2$ and $P_5 = Q_5$. Prove that $P = Q$.*

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