

UBC MATH CIRCLE 2024 PROBLEM SET 9

Problem 1. Prove that the equation $x^2 - 5y^2 = 1$ has infinitely many solutions in integers.

Problem 2. Let P be a polynomial with integer coefficients such that $n \mid P(2^n)$ for every positive integer n . Prove that $P \equiv 0$.

Problem 3. A set X consisting of n positive integers is called good if the following condition holds:

(*) For any two distinct subsets $A, B \subset X$, the number $\sum_{a \in A} a - \sum_{b \in B} b$ is not divisible by 2^n .

Given $n \in \mathbb{N}$, find the number of good sets of size n , all of whose elements are strictly less than 2^n .