TILINGS

Mathematical Methods behind Stunning Symmetries

thanks to www.scienceu.com

Tilings around us

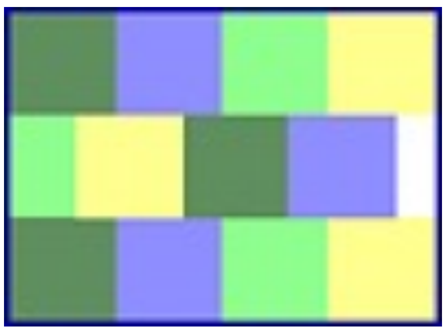
Walls are tilings

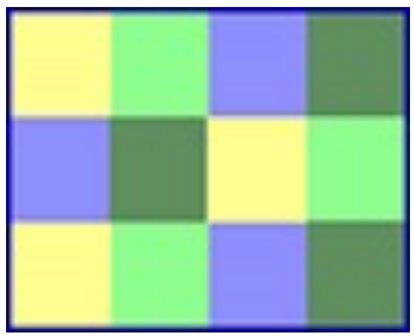
Beehives are tilings

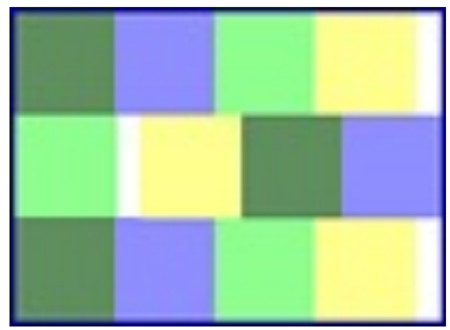
Tilings are tilings

What is a tiling?

A way of covering a flat surface with smaller shapes so there are no gaps or overlaps

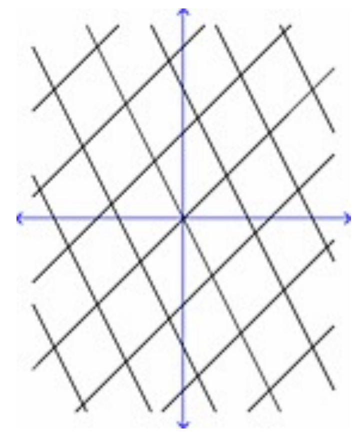






Some definitions

We'll study plane tilings where the surface is the 2-dim plane and the smaller shapes are a collection of 2-dim shapes called tiles



Making tilings

To recreate a tiling we need to decide how many different tiles we need. We say two tiles are congruent if they are the same size and shape (but colour can be different)

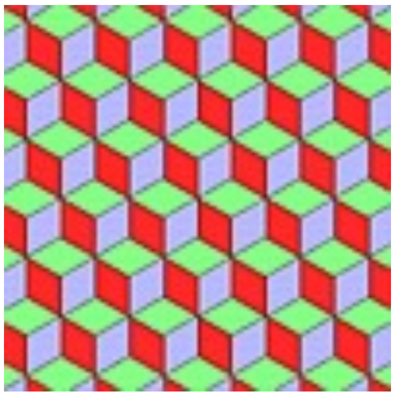
Not congruent

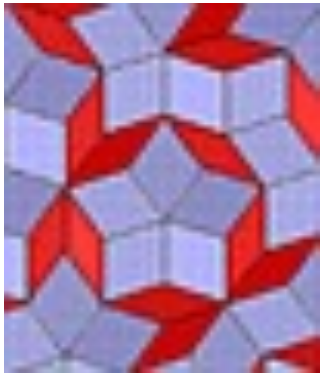
These are not congruent because they are different shapes

These are not congruent Because they are different sizes

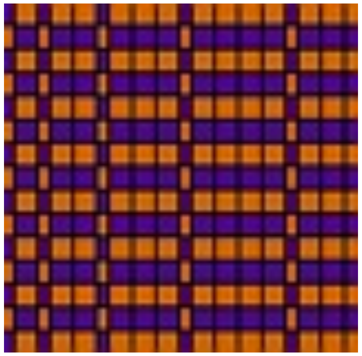
Prototiles

If we have some tiling T then the minimum set S such that every tile is congruent to one tile in S is the generating set of prototiles



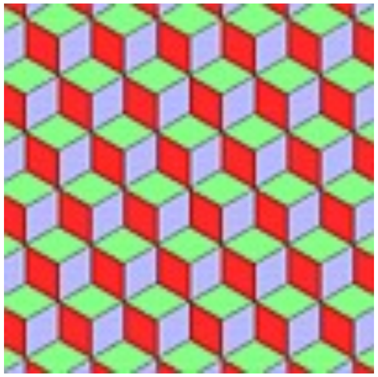


Making nice tilings



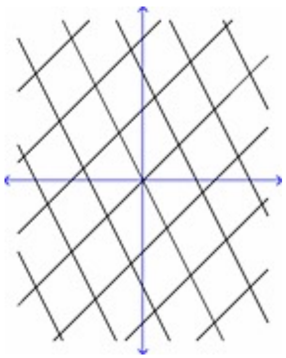
Repeating tilings =
repeats in one direction

Periodic tilings =
repeats in two directions



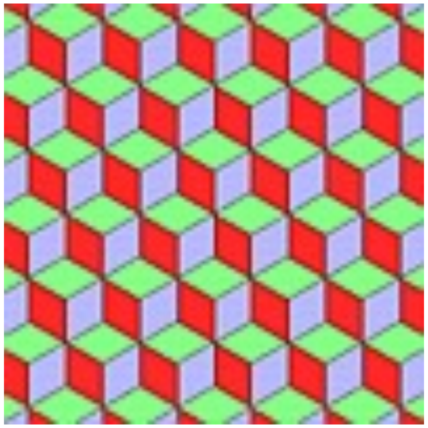
Test for periodic tilings

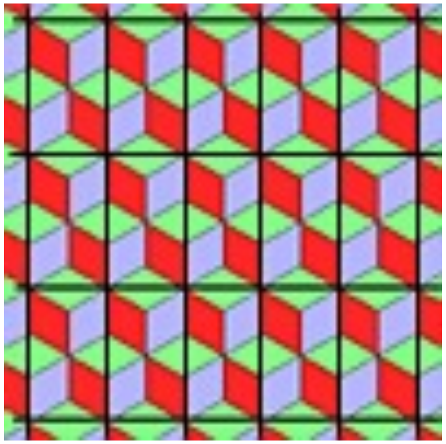
A lattice is a grid consisting of two sets of evenly spaced parallel lines

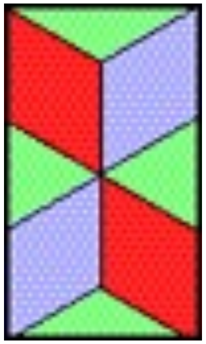


A tiling is periodic if we can find a lattice to lay over our tiling so every parallelogram in the lattice is the same

Trying out the test

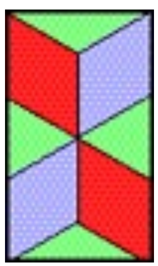


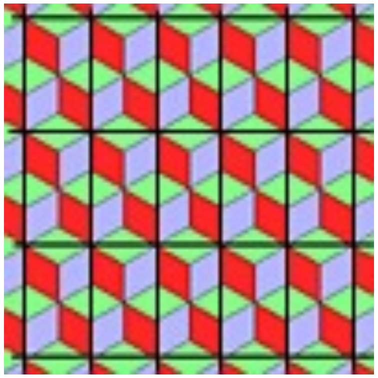


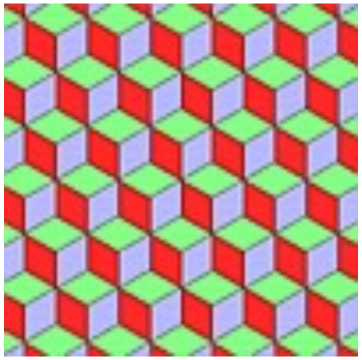


Basic units

We'll call the parallelogram a basic unit and make the periodic tiling by translating copies and pasting

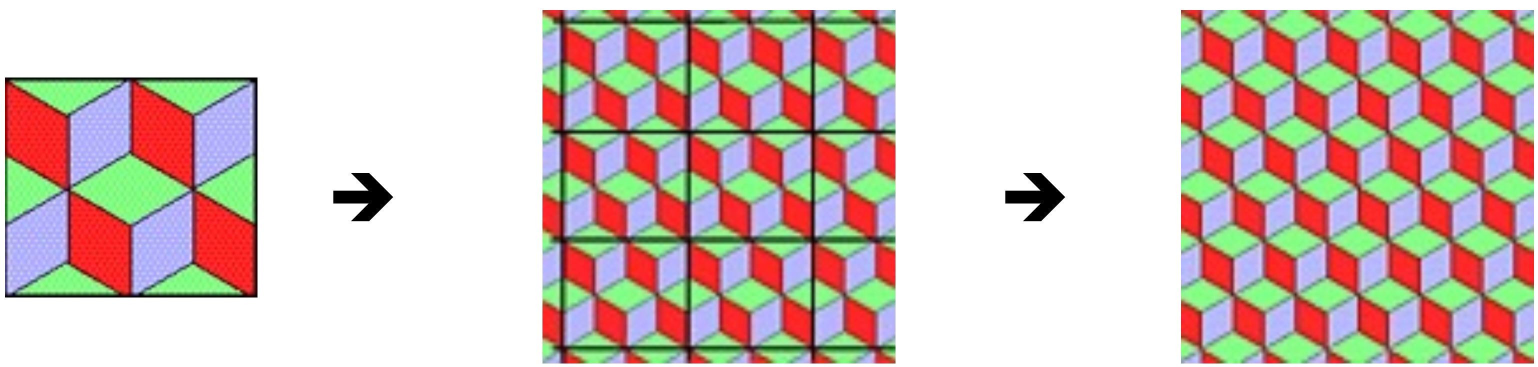






Is the basic unit unique?

NO here is another…



Symmetry in periodic tilings

A figure in the plane is symmetric if you can pick it up, move it around, put it down again so it looks like it hasn't moved

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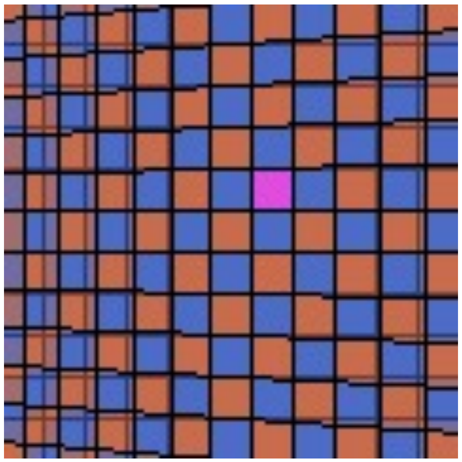
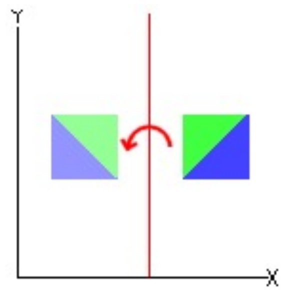
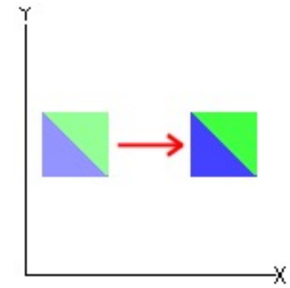


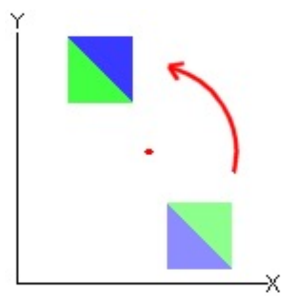
Isometries: kinds of moves



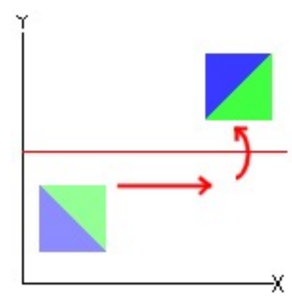
Reflection



Translation



Rotation



Glide reflection

Symmetry group

The specific isometries that fix your tiling are called symmetries and the collection of all its symmetries is the symmetry group

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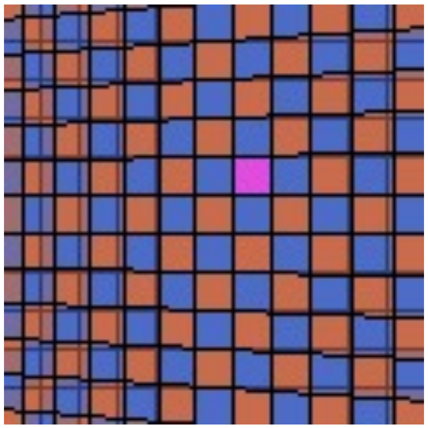
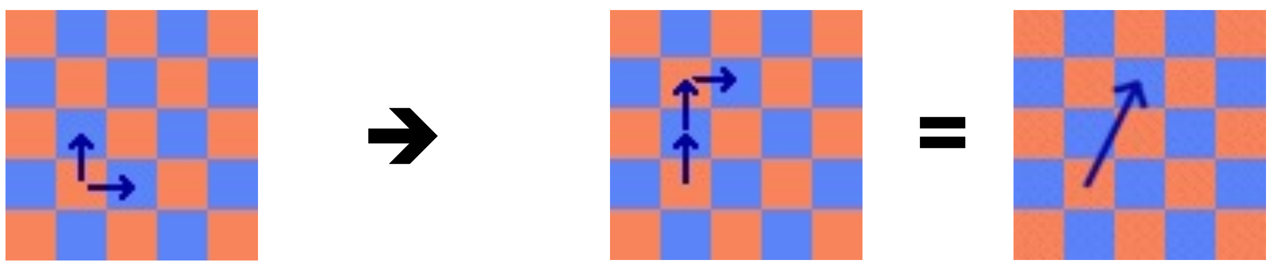


Generators of a symmetry group

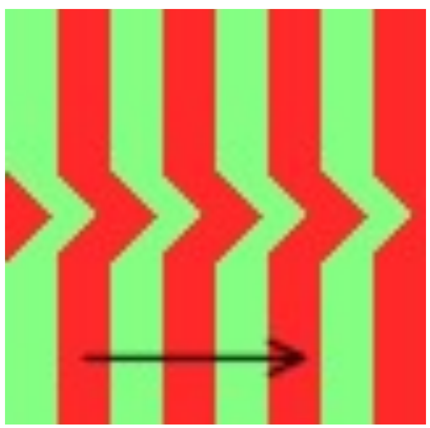
When we combine two symmetries we get another, so symmetry groups are big

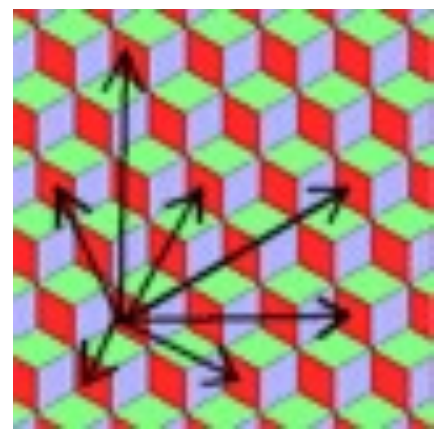


The basic ones are called generators

Wallpaper tilings

If you can slide the tiling in any direction so it eventually looks like it hasn't moved



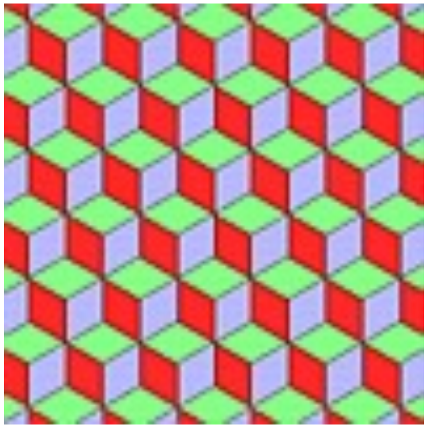


Fundamental domains

Q: can we use tiles and symmetries for a better description than basic units?

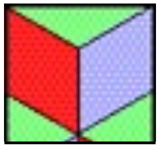
A: Yes! Make a list of all the symmetries that are generators. Now find a small piece of tiling that always moves when the generators are applied. Make the tiling by moving this fundamental domain

Making wallpaper



has generating set 180 degree rotation and horizontal translation

and fundamental domain



Making different wallpaper

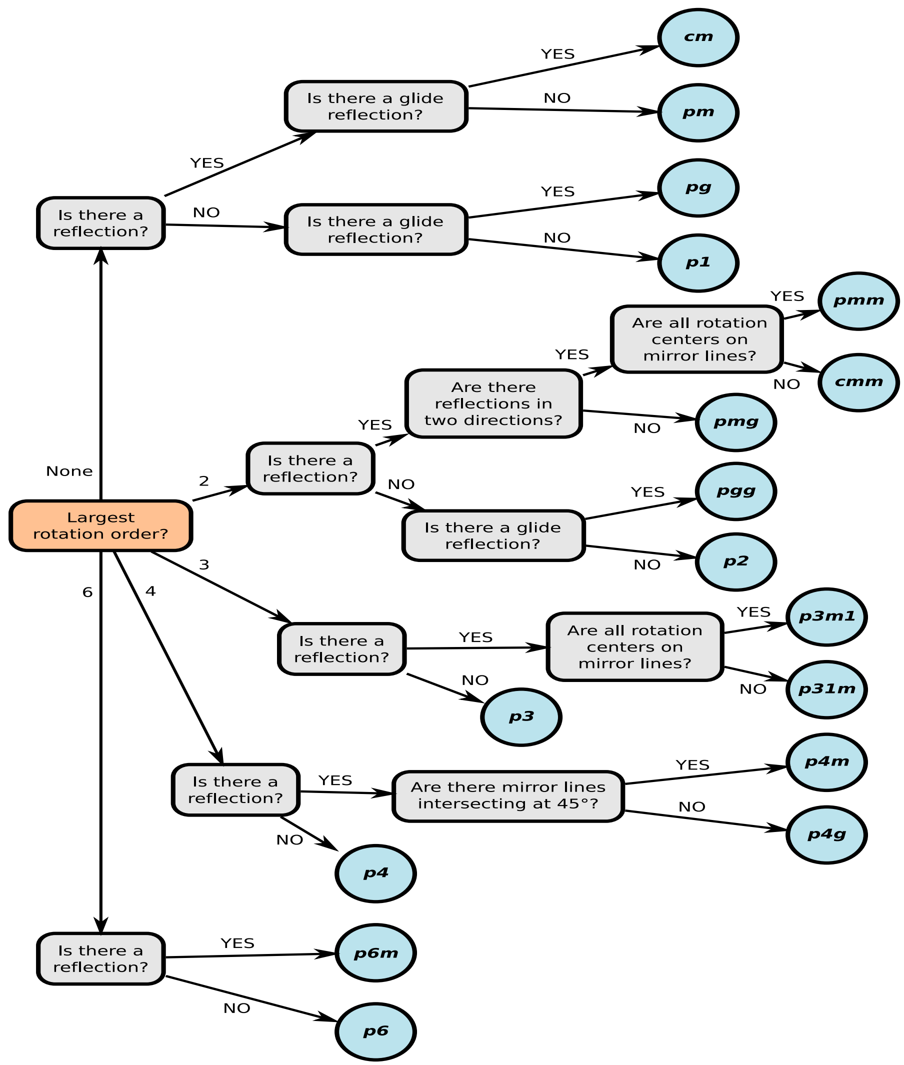
Can we count all symmetry combinations that will give a wallpaper tiling?

Making different wallpaper

Can we count all symmetry combinations that will give a wallpaper tiling?

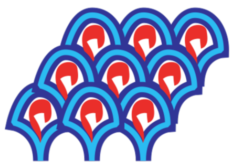
YES! There are only 17

Identifying wallpaper

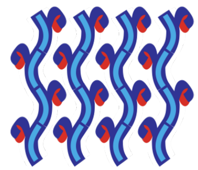


Identifying wallpaper

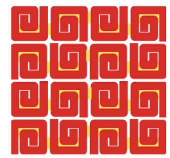
p1



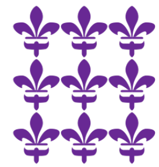
pg



pgg



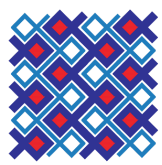
pm



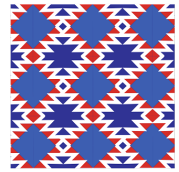
cm

cmm

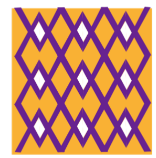
pmg



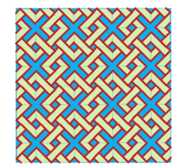
pmm



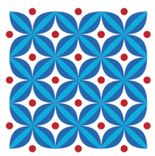
p2



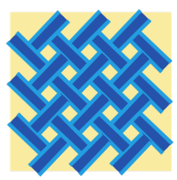
p4



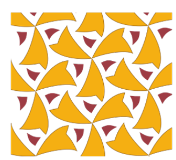
p4m



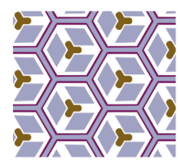
p4g



p3

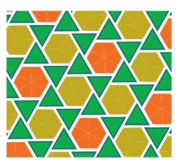


p3m1



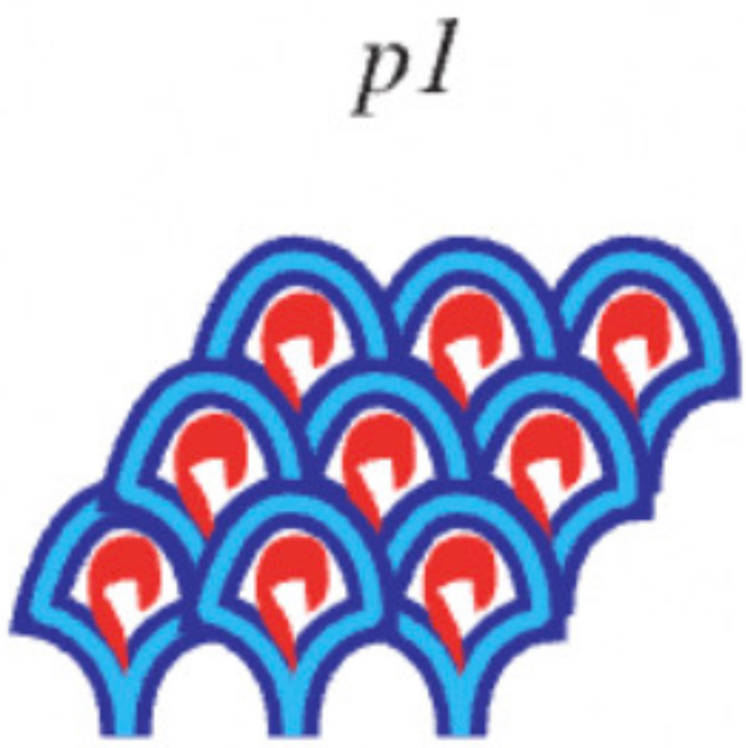
p31m

p6

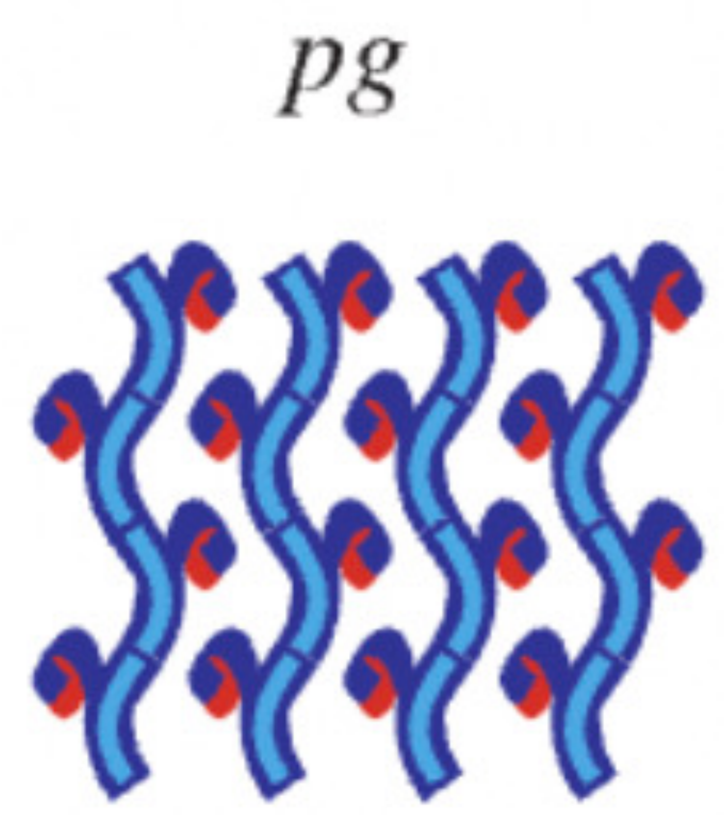


p6m

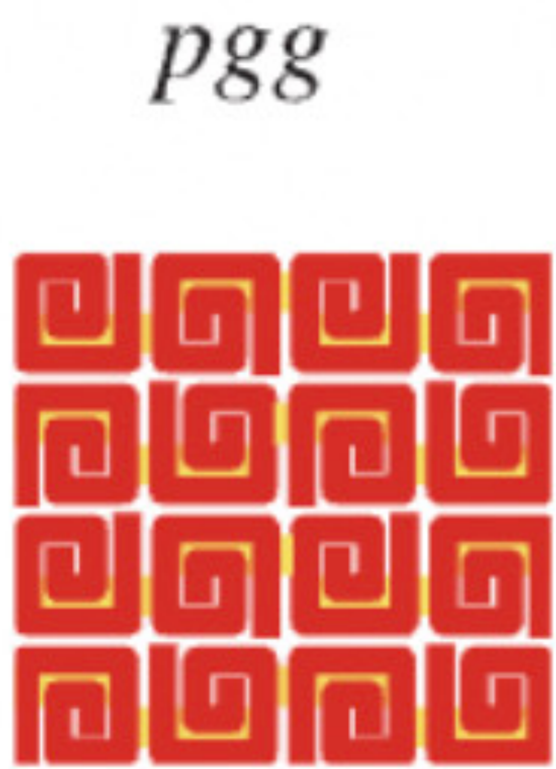
p1: translation in any direction



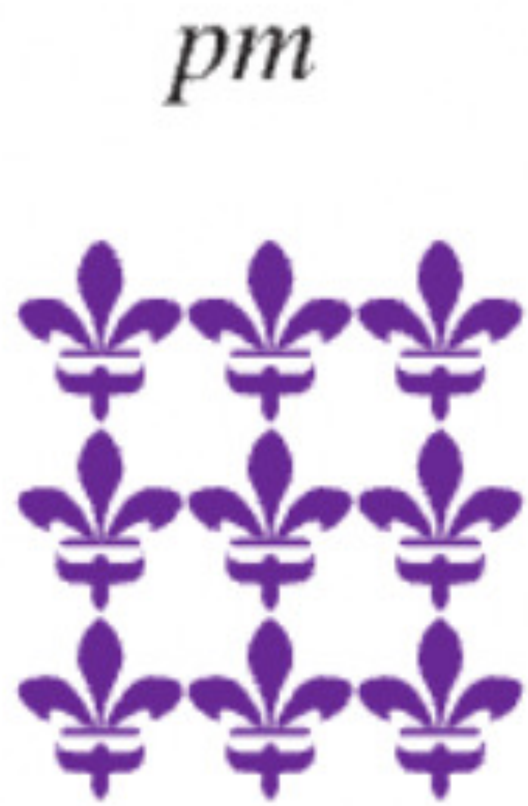
pg: glide reflection



pgg: 2 perpendicular glide reflections + 180 degree rotation

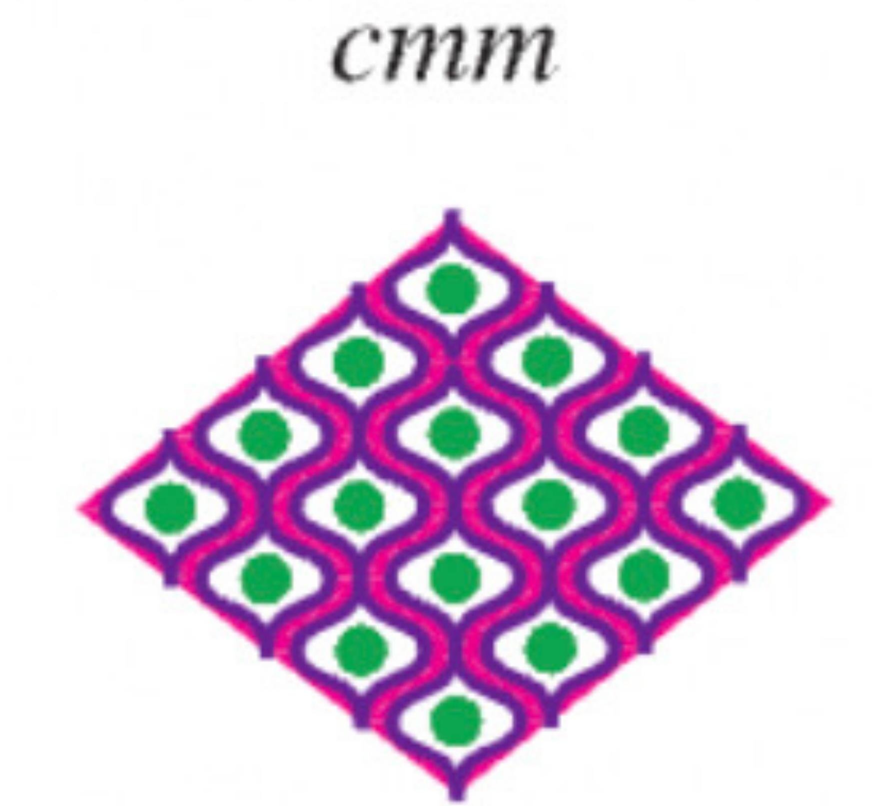


pm: axis translation + reflection

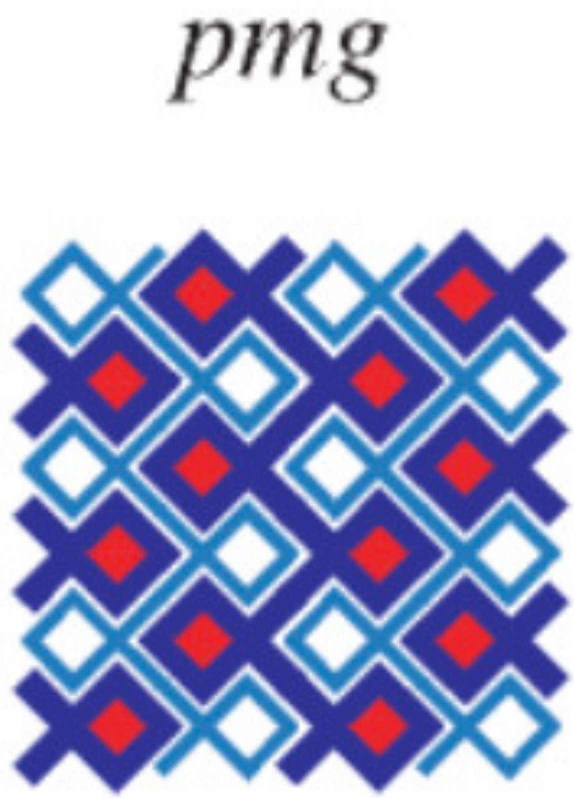


cm: glide reflection + reflection in parallel axis

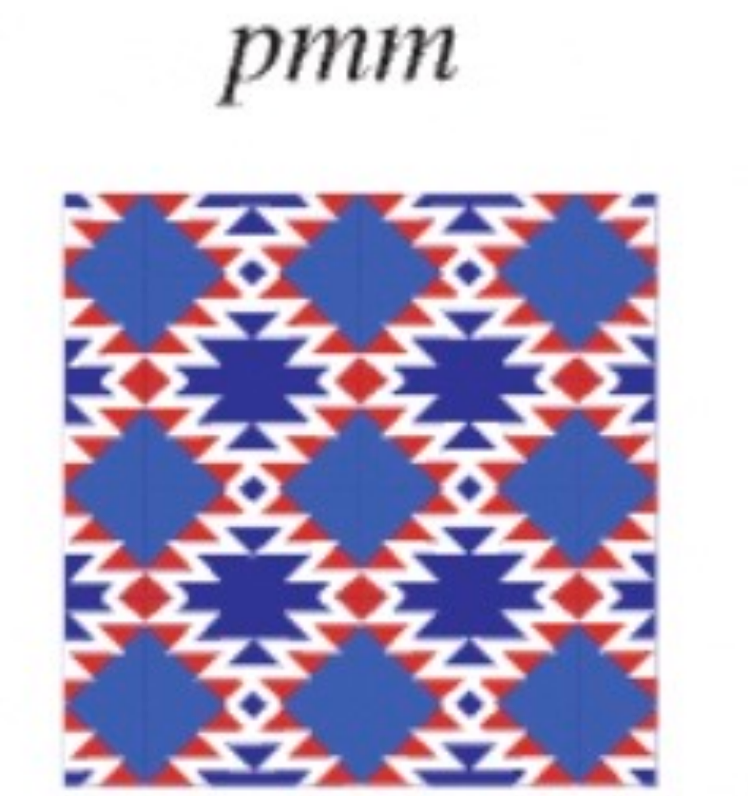
cmm: 2 perpendicular reflections + 180 degree rotation



pmg: reflection in 1 axis + 180 degree rotation



pmm: 2 perpendicular reflections

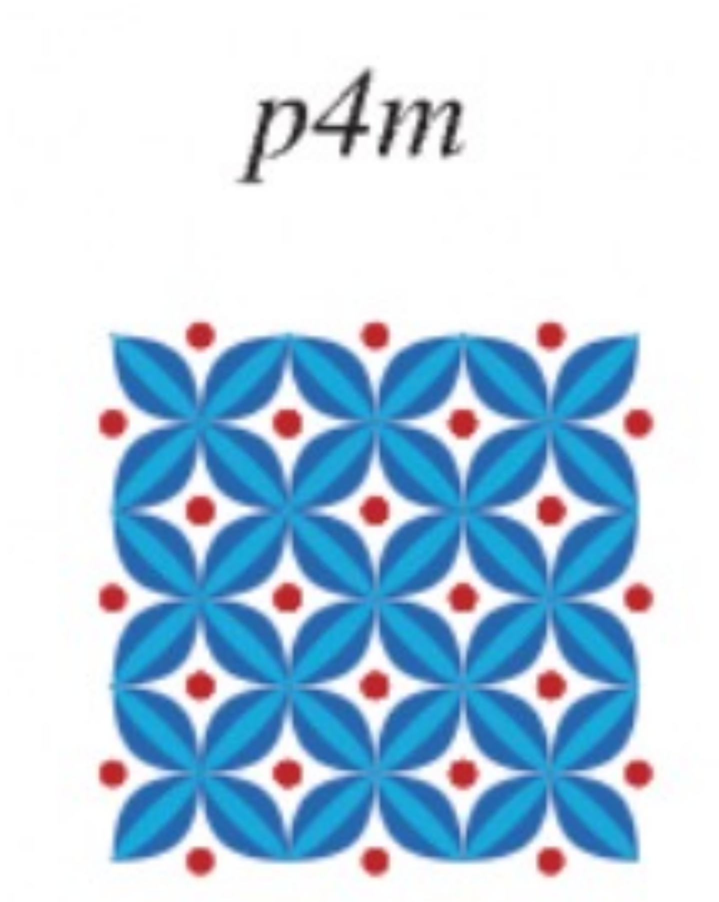


p2: translation + 180 degree rotation

p4: 90 degree rotation

p4

p4m: 90 degree rotation + 4 reflections



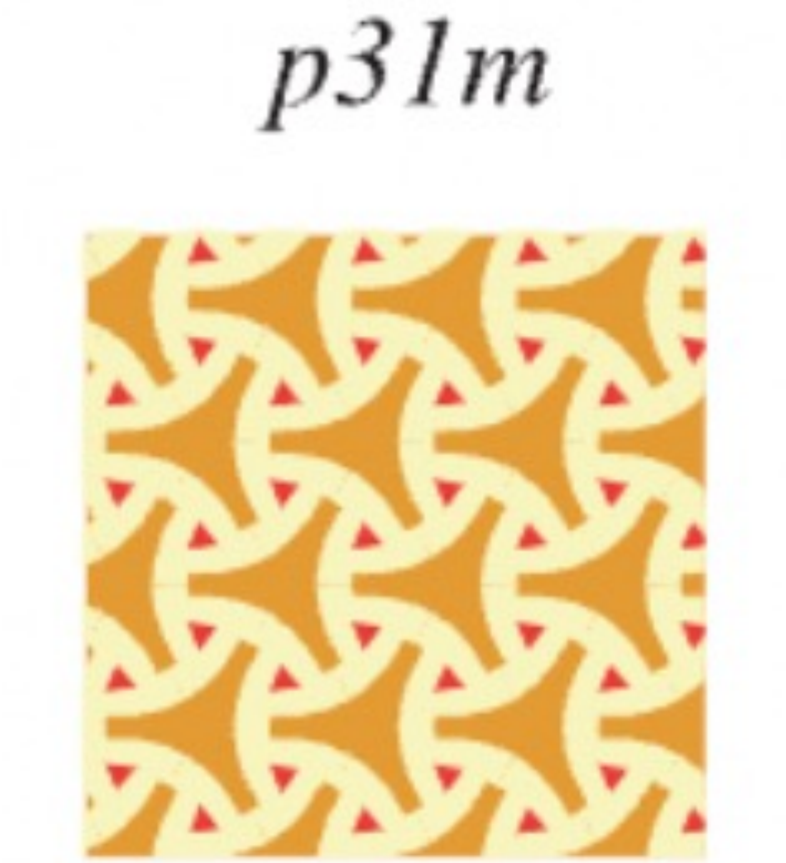
p4g: 90 degree rotation + two perpendicular reflections

p3: 120 degree rotation

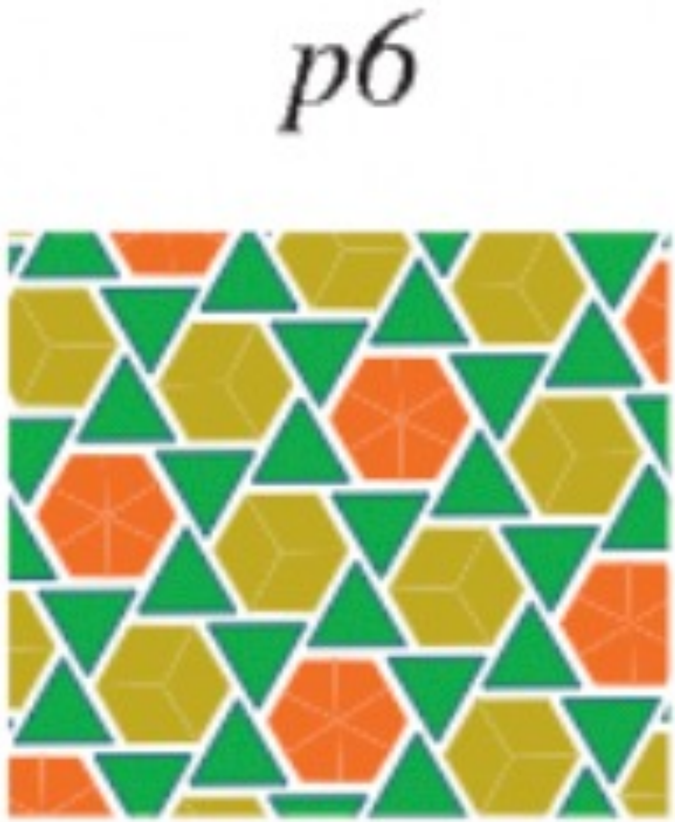
p3

p3m1: 120 degree rotation + reflection

p31m: 120 degree rotation + different reflection



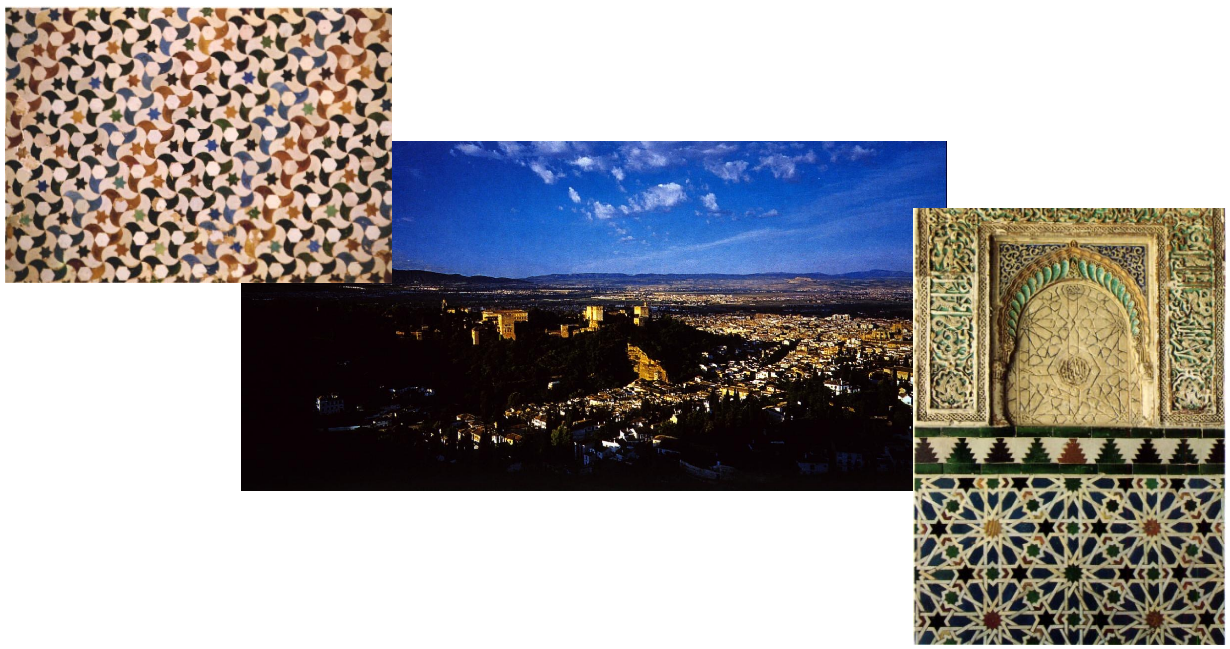
p6: 60 degree rotation



p6m: 60 degree rotation + reflection

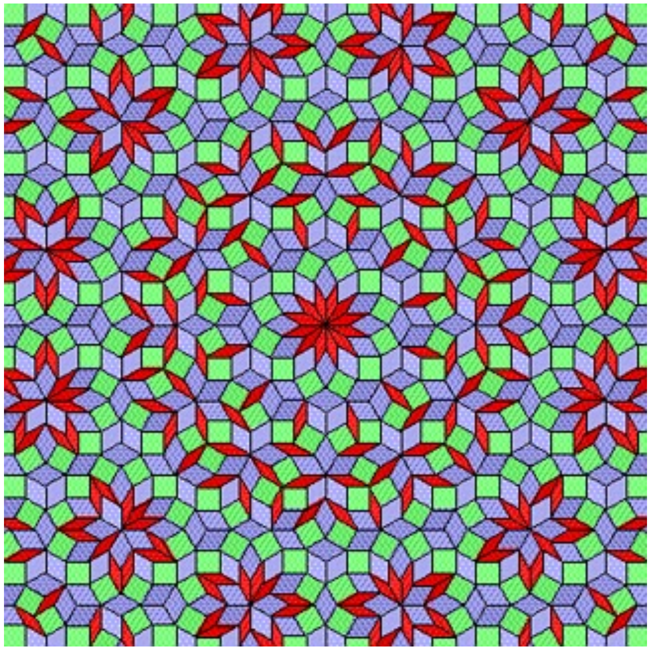
Open problem

The Alhambra in Spain is reputed to have all 17 wallpaper tilings somewhere on its walls…



Final question

Find a basic unit or fundamental domain?



Thank you!